

# Is the secrecy of the parametric configuration of slot machines rationally justified? The exposure of the mathematical facts of games of chance as an ethical obligation

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## Abstract

Slot machines have gained high popularity despite a specific element that could limit their appeal: non-transparency with respect to mathematical parameters. The PAR sheets, which expose the parameters of the design of slot machines and probabilities associated with the winning combinations, are kept secret by game producers, and the lack of data regarding the configuration of a machine prevents people from computing probabilities and other mathematical indicators. In this article, I argue that there is no rational justification for this secrecy for two reasons: one psychological and the other mathematical. For the latter, I show that mathematics provides us with some statistical methods of retrieving the missing data, which are essential for numerical probability computations in slots. The slots case described herein raises the problem of exposing the parametric configuration and mathematical facts of any game of chance as an ethical obligation.

## Résumé

Les machines à sous jouissent d'une grande popularité en dépit d'un élément particulier qui pourrait en limiter l'attrait : l'absence de transparence en ce qui touche les paramètres mathématiques qui déterminent leur fonctionnement. Les fabricants de jeu ne divulguent pas les documents précisant les paramètres de conception des machines à sous ainsi que les probabilités associées aux combinaisons gagnantes. Cette absence de données sur la configuration des appareils empêche quiconque de faire un calcul des probabilités et d'autres indicateurs mathématiques. Dans le présent article, j'avance qu'aucune explication rationnelle ne justifie le maintien du secret en me fondant sur deux arguments, l'un relevant de la psychologie et l'autre, des mathématiques. En ce qui concerne le second, je démontre qu'il existe des méthodes statistiques qui permettent de récupérer les données manquantes, essentielles au calcul des probabilités

numériques. Le cas des machines à sous soulève le problème de la divulgation des configurations paramétriques et des principes mathématiques des jeux de hasard en tant qu'obligation d'ordre éthique.

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### **Popularity of Slot Machines and Their Particularities in Regard to Exposure of the Parametric Configuration**

There is a relation between the popularity of a game and excessive gambling in it, which is also visible on any pathway model to problem gambling in its first phase, namely, *ecological factors* involving *availability* and *accessibility* (Blaszczynski & Nower, 2002). For decades, the slots game has remained one of the most popular games of chance, and the main elements that contribute to its position—from the perspective of both a problem gambler and a non-problem gambler—are as follows:

1. Variety. Players may choose from among a wide palette of games with respect to rules, parameters, and design according to their profile, goals, or even hobbies (Griffiths, 1999; Wood, Griffiths, Chappell, & Davies, 2004).
2. Privacy. Slots games, played either in a public place or in front of the computer, assume a private space with minimal exposure in which there is only the player and the machine (Parke & Griffiths, 2002), unlike a roulette table, for example, where you place your bets together with other players.
3. Attractive design. From the graphics of the symbols to the design of the interface, all is sparkling and brightly colored (Wood et al., 2004).
4. Brevity. A game timeline is short, lasting a few seconds from credit insertion to the stop of the spin, and players prefer a high number of games played within a time unit (Griffiths, 1999).
5. Illusions and sensations. Multiple design and configuration features distort players' perceptions (e.g., just missing the jackpot by one symbol can cause players to think a big win is imminent; use of a "stop" button can foster the illusion of control over machine outcomes; Griffiths, 1999).

Slot machines gained and maintained this popularity despite some specific elements that could limit their appeal, particularly:

1. Non-transparency. Players do not know the configurations of the machines they play, as this information is not exposed. Blackjack players know the composition of the decks in play, roulette players know the numbers on the wheel, lottery players know the numbers from which the winning line is drawn, and so on. Slots remains the only game in which players are not aware of the essential parameters of the game, such as the number of stops of the reels, the number of symbols, and the distribution of the symbols on the reels.

2. Prevention from odds estimation. The lack of data regarding the configuration of a machine prevents people from computing the odds of winning and other mathematical indicators.

The so-called PAR sheets (Probability Accounting Reports)) – documents that expose the weighting of the reels, some of the probabilities associated with the winning combinations, and other statistical indicators – are kept secret by game producers.

Practical questions arise about whether slot machine popularity would decrease and the slot player's behavior would be influenced if these hidden probabilities were exposed, and whether there exists a rigorous method through which qualified persons studying slot games (e.g., statisticians, applied mathematicians, programmers) can retrieve the parameters of the configurations of slot machines in order to generate their own PAR sheets for the players. I answer the first question in the sections *The Psychological Argument Related to Competition* and *The Psychological Argument Related to Players*, and the second question in the section *Statistical Methods for Estimating the Parameters of the Configuration of a Slot Machine*.

### **The Parametric Configuration of Slot Machines as a Base for the Probabilistic Models for Slot Games**

Although less difficult than other games with respect to the ease of probability calculus (compared with card games, for example), slot games still fall into that category of games of chance for which the probability computations cannot be conducted and performed by the average player, as such computations require medium to advanced knowledge and skills in probability theory. Therefore, the final probability results for these games, in the form of numerical probabilities or formulas (or software programs/applets using those formulas) ready to be computed by inserting the parameters of the specific game design and of the event to be measured, can be delivered to gamblers only by qualified persons.

For the applied mathematician, the hardest task in establishing the mathematical model for the probability calculus in slots is the optimal categorization of slot games so as to be able to obtain general probability formulas with variables describing all possible parametric designs of the machines and all winning events. This difficulty is due to the wide variety of existing and possibly forthcoming slot games with respect to their parametric configuration, which consists of configuration of the reels and configuration of the display.

#### **Configuration of a Reel**

The configuration of a reel refers to the distribution of the symbols over the stops of that reel and the arrangement of the symbols:

Denoting by  $t$  the number of stops and by  $p$  the number of distinct symbols  $S_1, S_2, \dots, S_p$  on the reel, and denoting by  $c_i$  the number of symbols  $S_i$  on the reel ( $1 \leq i \leq p \leq t$ ), then the vector  $c_1, c_2, \dots, c_p$  is called the *distribution of the symbols*  $S_1, S_2, \dots, S_p$  on the reel, also known as the *weighting* of a reel. Each reel has its own distribution of symbols.

Given the number of stops  $t$ , the number of distinct symbols  $p$ , and a distribution  $c_1, c_2, \dots, c_p$  of the symbols on a reel, there are several ways of arranging those symbols on the stops of that reel. Any function  $a$  from the set of stops to the set of distinct symbols, such that  $|\{x|a(x)=S_i\}|=c_i$  for any  $i$  from 1 to  $p$  (that is, the number of stops having assigned symbol  $S_i$  by function  $a$  is  $c_i$ ), is called the *arrangement of the symbols* on the reel.

The distribution and arrangement of the symbols on each reel identify a slot game and are determined by the game producer.

The configuration parameters are essential for the probability computations. The numbers of stops of the reels and the symbol distributions on the reels stand as variables for any general formula for the probability of a winning event defined on a payline made of independent stops (stops belonging to independent reels, that is, a payline that crosses over the reels without overlapping them). The symbol arrangements on the reels count toward the probability computations of winning events defined on paylines that hold stops of the same reel, assuming the arrangement is known.

Consequently, for ethical reasons, the parameters of the configuration of a slot machine should be present in any technical sheet describing that machine—for either internal or external use—and in its associated PAR sheet, along with the computed probabilities of the winning combinations and other statistical indicators, which I advocate for later in this article.

### **Configuration of the Display**

The configuration of the display refers to the shape and structure of the set of windows showing the visible symbols of the reels, which produce the outcome of the game, as well as the shape, length, and position of the paylines. All of these properties are described mathematically through geometrical and topological properties of those sets (a rigorous model for the configuration of the display is a rectangular grid in which the lines are defined as discrete paths linking neighboring points).

From the whole configuration of a display, the length of a payline is the only parameter that counts toward the probability computations for events related to that line, regardless of its shape or other properties; however, for more complex events defined on several paylines (for instance, of the type “*a specific winning*

combination of symbols on any payline from a given group of paylines”), particular properties and parameters of that group, such as those related to *intersection* and *independence*, also count.

The mathematical model of the configuration and the probabilistic models based on it are idealized models that do not represent all types of slot machines on the market.

### Variables for the General Formulas of Probability and Expected Value

Under the assumption that the reels spin independently (either physically or virtually, in the sense of probabilistic independence), we can obtain general formulas for the probability of the various winning events related to one or several paylines, having as variables the parameters of the configuration of the slot machine described in previous sections.

Most slot machines do not have the same number of stops on their reels, nor the same distribution of symbols on them (unfortunately, for the ease of computations!). Yet we can assume the same number of distinct symbols on each reel (denoted by  $p$ ) through a convention: If a symbol does not appear on a reel, we could simply take its distribution on that reel as being zero. A blank is considered as a distinct symbol within the mathematical model.

We distinguish two possible types of slot machines with regard to the parametric equality of their reels:

Type A – All reels have the same number of stops and the same distribution of symbols; each symbol  $S$  has the same distribution (number of instances)  $c_S$  on the  $t$  stops of each reel; denote by  $c_i$  the distribution of symbol  $S_i$  on each reel ( $1 \leq i \leq p$ );

Type B – The reels have different numbers of stops  $t_j$  and each symbol has different distributions on the stops of the reels, denoted by  $c_i^j$  ( $c_i^j$  is the distribution of symbol  $S_i$  on reel number  $j$  ( $1 \leq i \leq p$  and  $1 \leq j \leq n$ ), where  $n$  is the number of reels).

Call the two situations case A and case B.

Given a specific symbol  $S_i$ , the probability of  $S_i$  occurring on a reel after a spin is

$$q_i = \frac{c_i}{t} \text{ in case A and } q_i^j = \frac{c_i^j}{t_j} \text{ in case B}$$

where  $j$  is the number of that reel. The numbers  $q_i$  and  $q_i^j$  are the *basic probabilities* in slots.

For an event  $E$  related to an independent-stops line of length  $n$ , the general formula of the probability of  $E$  is as follows:

$$P(E) = \frac{F(E)}{t^n} \text{ in case A and } P(E) = \frac{F(E)}{\prod_{j=1}^n t_j} \text{ in case B,} \quad (1)$$

where  $F(E)$  is the number of combinations of stops favorable for the event  $E$  to occur. For winning events  $E$  defined in a cumulative manner (that is, through numbers [quantities] of specific symbols necessary for the payline to hold, regardless of their position on the payline), which is the case for most slot games,  $F(E)$  has a polynomial expression, being a function of  $t$  and  $c_i$  in case A or of  $t_j$  and  $c_i^j$  in case B ( $1 \leq i \leq p$  and  $1 \leq j \leq n$ ). For instance, if the event  $E$  is in particular *exactly one instance of a specific symbol S*, formula (1) is written as follows:

$$P(E) = \frac{nc_s(t-c_s)^{n-1}}{t^n} \text{ in case A and } P(E) = \frac{\sum_{i=1}^n c_s^i \prod_{\substack{1 \leq j \leq n \\ j \neq i}} (t_j - c_s^j)}{\prod_{j=1}^n t_j} \text{ in case B} \quad (2)$$

Still, in particular, for an event  $E$  expressed through the number of instances of each symbol on a payline in case A, formula (1) becomes the classical formula of probability in a polynomial field:

$$P(E) = \frac{n!}{a_1! a_2! \dots a_p!} (q_1)^{a_1} (q_2)^{a_2} \dots (q_p)^{a_p} \quad (3)$$

where  $a_1$  is the number of instances of  $S_1$ , and so on, and  $a_p$  is the number of instances of  $S_p$  ( $a_1 + a_2 + \dots + a_p = n$ ). Parameters  $a_1$  to  $a_p$  characterize the winning combination, while  $q_1$  to  $q_p$  characterize the configuration of the reels.

Formula (3) is used when the winning event is defined through an exact distribution of all symbols on the payline (even if some of them do not appear in the winning combination, having the distribution zero).

Particularizing in case B, consider the event  $E$  as *exactly  $m$  instances of a specific symbol S* ( $m \leq n$ ). Formula (1) becomes in this particular case:

$$P(E) = \sum_{1 \leq i_1 < i_2 < \dots < i_m \leq n} \prod_{j \in \{i_1, i_2, \dots, i_m\}} q_S^j \prod_{\substack{1 \leq k \leq n \\ k \neq i_1, i_2, \dots, i_m}} (1 - q_S^k) \quad (4)$$

where  $q_S^i$  is the basic probability of occurrence of symbol  $S$  on reel number  $i$ .

The general formula (1) holds for simple events related to one payline. For more complex events such as unions of winning events on one or several paylines, formula (1) is used along with other properties of probability and methods of

approximations for obtaining applicable (although overloaded) formulas for the probability of those events (Bărboianu, 2013a).

From the expression of the probability formulas presented, one can see that these are functions of  $n$  (the length of the payline),  $t$ , and  $c_i$  in case A, or of  $t_j$  and  $c_i^j$  in case B ( $t$  or  $t_j$  are the numbers of stops of the reels;  $c_i$  or  $c_i^j$  are the distributions of the symbols on the reels), and of other variables describing the event to be measured. For events related to several paylines, other variables describing the event also appear in the formulas (the number of lines of the group, cardinalities of the intersections of those lines, etc.). The formulas can also be written in terms of basic probabilities  $q_i$  or  $q_i^j$ , which can replace the  $c$  and  $t$  variables as ratios between them.

The same variables will also appear in the general formulas of the expected value of the slots bets, along with the payout rates from the payout schedule of the game.

I present this overview of the basic elements of the mathematical model necessary for probability calculus in slots, as well as a few general results, to emphasize the necessity of having the data describing the configuration of a slot machine as inputs for the probability computations. By applying mathematics within a general model, we can obtain only general formulas for probability and expected value. However, these formulas or even associated tables of values are useless for the player without final numerical computations for a given game, and these can be performed only if we are provided in advance with the numerical parameters of the configuration of that game.

### **The Secrecy of Slots PAR Sheets: Facts, Justifications, and Implications**

The secrecy of game producers on PAR sheets is verified by the lack of availability of these files. One can see unsuccessful PAR sheet requests from slot players by browsing their forums, while game researchers can obtain them only through legal intervention.

Since 2007, game researchers have obtained the PAR sheets of some slot games through the Freedom of Information and Protection of Privacy Act in Canada (Harrigan & Dixon, 2009). Most of these PAR sheets became public after researchers studied them; others were also available before 2007 through other channels (see Wilson 2004a, 2004b, 2004c, 2004d, 2004e). Still, these are the PAR sheets of a miniscule part of all slot games on the market. *Slot Tech* magazine has published articles dedicated to PAR sheets; however, they are limited in the number of games covered and in that the description is adapted to the audience of this magazine, that is, technicians servicing the machines (Harrigan & Dixon, 2009).

Browsing the appeal decisions of the Information and Privacy Commissioner (IPC) in Ontario, Canada, with respect to PAR sheet requests, one can see that game producers who declined requests invoked the exemption set forth for scientific and technical information, through one or more of the facts that PAR sheets contain information routinely considered to be trade secrets in the gaming industry and consist of mathematical formulas and equations developed by their engineers. They further claim that information provided on PAR sheets significantly compromises their competitive position and interferes significantly with the contractual obligations of the company (IPC, 2009, 2010).

Major slot companies have brochures in circulation presenting their games to players, more promotional than informational, in which mathematical aspects of the games are barely touched upon. For instance, a brochure by International Game Technology (IGT), called *Introduction to Slots and Video Gaming*, has a section titled “Slot Math,” that presents three examples of games, each from a large category (three-reel, four-reel, and five-reel slot machines), for which the following configuration parameters and mathematical facts are shown: number of stops of each reel, the distribution of the top-award symbol (that symbol triggering the jackpot in a line-up combination) on each reel, hit frequency of the top-award combination, and the overall hit frequency of any winning combination. Summarizing the math section, it provides the player with the numbers of stops of the reels, distributions of one symbol on the reels, and two probabilities (one for the top-award combination and the other for any winning combination) for three specific games; there are no distributions of the other symbols, no probabilities for other winning combinations, and no expected value, which is a relevant indicator of the practical risk over the long run. In addition, the use of the term *symbol combination* in probability context is confusing because with this term, the winning combinations shown on the payout panel are defined; the combinations involved in the probability computations are *combinations of stops* (holding those symbols), not combinations of symbols. To “explain” the brevity of their math section, the editor wrote in the introduction (IGT, 2009):

[...] One such tool, par sheets, can be complicated to understand. However, investing the time learning to read them is time well spent. They offer important information for optimizing the revenue for each machine, as well as offering data for technicians. In this section, we provide examples of simple slot math that is found on par sheets for three types of games – spinning reel and video reel slots, video poker, and bonus games. These equations represent the most basic operations only. For more detailed information, please ask a gaming representative or attend training classes. Par sheets for all IGT games [...] are available online at [...]

Going to the webpage indicated as providing PAR sheets for all IGT games, I found a link labeled “PAR sheets” hidden on a third-level page, for which the browser returned a 310 error when accessed. The problem of accessing that section might



have been temporary; however, there is a disclaimer on that page that certain areas of the site are secured and require an active member account. I made no further investigation of whether opening such an account leads unconditionally to a positive answer regarding a PAR sheet request.

Given all the facts presented above, it is clear that slot producers are strongly reticent in exposing the parametric configuration of their games.

### **The Psychological Argument Related to Competition**

Coming now to the justification of this reticence, the slot producers' reasons for declining PAR sheet requests, shown in the IPC's appeal decisions, seem to be judicially formal rather than factual. I present the following arguments for this claim:

- The trade secret and intellectual ownership reasons fail against the generality of the math formulas and equations. Although the parametric details vary from game to game, the mathematical results concerning probability, expected value, and other statistical indicators are just applications of general formulas that are publicly available in mathematics, thus common across all slot machines, and no individual or corporate body can claim ownership of such a pattern or formula. The argument also holds if talking only about the protection of the parametric configuration. Protecting a certain finite sequence (combination or arrangement) of symbols reverts to protecting the sequence of numbers that can be put in bijection with the former, since others may use it by just replacing the symbols with new ones through a new bijection on that sequence. But protecting a sequence of numbers falls within the same argument that mathematics is freely available.
- There are three possible reasons for competitive prejudice against competition, two of them coming from the situation in which another producer copies and uses the revealed parametric configuration: (a) the possibility of losing a share of the market to the infringer, (b) the unethical development of the infringing company, and (c) the exposure to bad publicity from a competitor or neutral entities. The arguments as to why these reasons fail are the following:
  - a. The infringing company would develop a slot machine different in its external (physical) design from the machine having the original parametric configuration and yet having this configuration in common with the original (because the entire machine of the original producer is patented, even though its parametric configuration in and of itself may not be). With the parametric configuration invisible, if the new machine is successful at and after launch, there may possibly be three types of elements responsible for this success, alone or in combination: its physical design, the marketing, and its statistical indicators.

The first two types of elements have nothing to do with parametric configuration, and so they do not apply to the original producer's

competitive-prejudice argument. If statistical indicators such as frequency of wins and payback percentage prove in time to be responsible for the new machine's success—which is very likely to happen—this is not a consequence of the use of that specific parametric configuration alone, but of that together with a given payout schedule. A producer can manipulate the game parameters, including the payout schedule, in unlimited ways so as to obtain the desired statistical indicators for the house. That is, if the goal is to have the payback percentage of another machine, a better alternative is to use a new parametric configuration yielding the same percentage instead of replicating one; besides, within the same goal, if copying the parametric configuration of the reels, the infringing company would also need to copy the payout schedule. While the parametric configuration itself may not fall within patent restrictions, parametric configuration plus payout schedule is likely to do so; therefore, the infringing company should expect the original producer to recognize its own payout schedule on the new machine and take legal action toward the potential (at that time) infringers.

- b. The infringing company would get a ready-to-use parametric configuration, possibly non-patented, which can be used in two ways: keeping the original statistical indicators, including payback percentage, or adding a new payout schedule and getting different statistical indicators. The former alternative assumes copying the entire game parameters (and avoiding the expense of the mathematical work, which compounds the unethical behavior), which may be protected as a whole. If this happens, the infringing company exposes itself to lawsuit and bad publicity, since the parametric configuration used can be discovered through statistical observation (see the next section) by the original producer. The latter alternative gives no rational motive for the infringing company to use a copied parametric configuration instead of a new one, since the math work must be done for finding a payout schedule that will yield the desired statistical indicators. In addition, using only the copied parametric configuration would again expose the infringing company to bad publicity when revealed through statistical observation.
- c. In the situation of bad publicity as a result of revealing a PAR sheet (which would have as an issue the fact that probabilities and/or statistical indicators are not favorable for the players), the producer would have the option of defending him- or herself with an acceptable answer: the fact that all existent slot games have similar figures attached, not much different from those exposed, while the producer at least showed them to the public (“I am not the bad guy here—just those who keep the PAR sheets secret”). Such an answer may turn the negative publicity into a good marketing strategy.

The hypothetical situations related to reasons (a) and (b) would apply to a start-up company in the role of the infringing company, as it is very unlikely that an established producer would risk his or her position just to avoid the expense of the math work.

Thus, the slot producers' justification for the secrecy of PAR sheets related to competition is insubstantial, as I argue above, even though some of them claim the opposite in their judicial litigations. The previous arguments (related to competition) are part of what I called at large *the psychological argument*. The possible justification exclusively related to their players remains to be considered. That would mean that they are afraid of losing players who see the PAR sheets of their games, and so the popularity of slots would decrease. I argue in the next section that such a claim is likewise insubstantial, and I propose further research to confirm what is hypothesized.

### **The Psychological Argument Related to Players**

The facts are as follows:

1. PAR sheets are kept secret by slot producers, with isolated and rare exceptions.
2. Slot players continue to play slots in the absence of information regarding parametric configuration, probabilities, and statistical indicators of the games, maintaining the popularity of these games.

When we talk about hiding a data sheet, it is the content that is hidden, and for a PAR sheet this means parametric configuration (numbers of stops and symbol weighting of the reels), probabilities for the prize-award combinations, other probabilities, frequencies, and other statistical indicators.

With respect to the content of a PAR sheet, by putting myself in both the producer's and the player's position, I see two possible reasons for the secrecy, one related to competition (hiding the parametric configuration and the statistical indicators) and the other related to players (hiding probabilities and statistical indicators). Further empirical study on slot players can confirm the existence of these two reasons and perhaps find others, which may be treated thereafter.

The former reason cannot change players' behavior with regard to willingness to play slots, as that is not related to any new unethical or fraudulent strategy of the producer against them—everything is the same as it was.

In treating the latter reason, I will focus on the content of the PAR sheet that is the most accessible (as knowledge) to gamblers and has the greatest impact on them. Among all of the mathematical data related to a game, the *probabilities* of the various winning events are the most important for a player with respect to the objective evaluation of winning/losing possibilities and general gaming knowledge, even though their influence might not lead to gaming decisions and changes in gaming behavior. This top status of probabilities is because all players, regardless of their level of mathematical knowledge, have a basic understanding (although many times distorted or misconceived) of the notion of probability according to its common classical definition (the ratio between the situations favorable for an event

to occur and the number of all equally possible situations) and a basic interpretation of it as a degree of belief in the occurrence of an event. Moving then to *expected value*, this notion (the mean of a discrete random variable) already requires a new level of mathematical education not available to most slot players. Thus, it can be hypothesized that players perceive that the main reason for the secrecy of PAR sheets (related to themselves) is to hide the odds/probabilities of winning. If further empirical research confirms this hypothesis, we shall arrive at the conclusion that slot players expect low to very low odds of winning without acquiring this information,<sup>1</sup> since there is no other reason for keeping the odds secret if they were high compared with other types of games on the market.

Now, fact 2, along with the previous conclusion, leads to the prediction that slot gamblers will not stop playing in the event of exposure of the PAR sheets of the games they play, since they already expect low to very low odds of winning. Other visible elements of the games keep them attractive and popular (Griffiths, 1999; Wood et al., 2004) despite their special status among games of chance (in respect to the exposure of their parametric configuration, which I discussed in the first section of this article). There is also an addictive component of the other elements of attractiveness, and slots addiction should be studied as a particular type of addiction, given the missing-parametric-configuration feature of the slot games.

Further empirical study of slot players can confirm this theoretical prediction of not quitting slots under the condition of low to very low odds of winning exposed through PAR sheets. Such studies could possibly be conducted together with those proposed earlier as stand-alone research. Until then, we have an example provided by lottery players whose behavior seems to confirm my prediction in the slots case.

### **The Lottery Example**

From all games of chance, the lottery offers by far the lowest odds of winning its top prizes, on the order of one to millions or tens of millions for the first prize, and one to tens or hundreds of thousands for the second prize, for common lottery designs.

With a history traced back to antiquity (B.C.) for its birth and to the 15<sup>th</sup>–16<sup>th</sup> century

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<sup>1</sup>I have excluded the statistical indicators from this analysis and focused on probabilities, also given the possibility of their being misinterpreted by non-math gamblers. A proper interpretation of the mathematical data of a game with respect to favoring the player takes into account both probabilities and statistical indicators based on expected value, not one category or the other individually. For instance, a payback percentage of over 90% can be seen as high and therefore favorable by (and for) a player. However, it is mainly the probability of the biggest win (along with the payout of that win) that yields this high payback percentage, and the calculation is a mean over the long run (infinity in mathematical terms). Since probability translates to frequency in the gaming experience, it will not be the same for the player if that winning event (balancing the computed payback percentage) occurs on average in a lifetime or less. Besides, payback percentage is an indicator usually exposed outside the PAR sheet.

for expansion, the lottery remains the most stable and respected game of chance; contemporaneous studies of this game recognize its popularity and the fact that no decrease in this popularity has been reported (National Impact Gambling Study Commission, 2004). Most lottery players play regularly and are aware of the very low odds of winning. Even though most of them might not know the exact figures, all have a clue about the size of these odds, knowing that they are very close to zero, because this information has spread widely enough in common communication between lottery players and through the media to become a proven well-known characteristic of a lottery. Even knowing that the winning odds are very low, lottery players still continue to buy tickets on a regular basis and the lottery has never lacked for business. Why, then, should slot game producers worry about slots doing otherwise, since the odds of winning at slots are generally higher than those of winning a lottery?

Some will be quick to point out the following elements in which the two types of games differ, at least with respect to financial expectations and players' options to estimate and manage these expectations:

- Prize amount. Lottery games offer first- to third-category prizes in amounts higher than similar prizes in slots games, and the high prizes somehow compensate for the low probabilities of winning with respect to the decision of quitting the game.
- Enhancing the probability of winning. Lottery games allow an increase in the overall winning probability<sup>2</sup> through buying several tickets or playing systems with several lines in unlimited numbers for the same draw, while in slots the player may enable only a limited number of paylines for such an increase.
- Game frequency. Slot players can spin the reels of a slot machine thousands of times in a day, whereas in lottery, players must wait several days for a new game.

In response, I would argue that these non-equivalences should be ignored, as follows:

- Prize amount. The objection argument assumes that the high prize amount is the dominant factor in the lottery player's behavior of playing against the minute odds of winning. My argument: If this factor is merely addictive, in parallel, the slot games also have their specific addictive and entertaining components, and the existent balance between lottery and slots is inclined toward the latter because of its generally higher odds of winning. If the high-prize factor has a practical side in the lottery player's mind, such as "someday these [high prizes] will make me rich," given the assumed basic knowledge of the probabilities involved, the player can estimate and face the overall probability of this fortunate event happening in a lifetime as very low and the invested money as serious, which eliminates the

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<sup>2</sup>"Overall" has the sense of a disjunction of winning events that is measured in probability, that is, "winning with line no. 1" or "winning with line no. 2" and so on in our lottery example. The increase in probability refers to moving from a single event to a disjunction of events that includes that single event. This convention applies in every instance where these terms are used in the article.

practical expectation, contradicts the assumption, and thus reduces the factor to its addictive component, which has been already addressed. (Such probability estimation is at hand for everyone, by taking an average playing lifetime, assuming a weekly play, and multiplying the winning probability by the total number of plays).

- Enhancing the probability of winning. My argument: This increase is actually limited by the player's available funds for the total price of the ticket(s), and therefore, increasing the probability of winning cannot change the size of this probability from very low or low (for a one-line ticket) to medium (for a multi-line ticket or several tickets). For example, assume a player buys 500 one-line tickets at \$2 each (a serious investment for one draw), or an equivalent line-system ticket, at a 6/49 lottery. The overall winning probability will increase a maximum of 500 times, leading to a maximum of 0.00003575 (from 0.0000000715) for the first-category prize and 0.0092 (from 0.0000184) for the second-category prize, which are very low and low, respectively (these maximums apply if the played lines are independent, in the sense of a restriction on the number of the common numbers of those lines; Bărboianu, 2009). There is another limitation in playing large numbers of lines once, coming from the distribution of the prize fund—the prizes are not given by a fixed payout schedule, but the lottery company assigns a certain prize amount for each prize category as a fixed percentage from the ticket sales of that draw. Therefore, investing large amounts of money for increasing the chances for a draw might result at some point in winning less than invested. On the other hand, in slots, the payout schedule of a machine is fixed for all games (spins), and the increase in the overall winning probability can be acquired not necessarily by enabling more paylines, but by running more spins: With the technical features of the slot game, a player can run hundreds of independent spins in a reasonable time, and so we get the equivalent effect as in the multi-line play in the lottery case with respect to the probability increasing.
- Game frequency. My argument: This conclusion was drawn based on the criterion of probability size for one game, under the assumption that basic probability knowledge is accessible to most gamblers. Game frequency is not a criterion for and does not change the argument of the conclusion, as it does not change the probabilities attached to a game; game frequency can be a criterion for other comparative analyses of the two types of games, for instance, regarding how deviation from the expected value is directly perceived and accounted for by the math-inclined player.

In my analysis, when talking about probabilities of winning, I considered the slots prizes as given only by the winning combinations of a single machine and I ignored the progressive jackpot prizes, because including them would just incline the balance more toward slots in the comparison between lottery and slots games with respect to the analyzed prediction of not quitting slots under the condition of PAR sheets exposure.

With the preceding counter-arguments, I conclude the lottery example and declare it as relevant within the psychological argument for arguing for the prediction of not quitting slots under the condition of PAR sheets exposure. There are, of course, other characteristic features (except those related to financial expectation) that distinguish the two types of games from each other and might be responsible for a different behavior of the slot players than of the lottery players. Further research is needed to confirm or refute that idea.

In the next section, I also present another argument, this time merely mathematical, for the insubstantiality of the secrecy of slots PAR sheets.

### **Statistical Methods for Estimating the Parameters of the Configuration of a Slot Machine**

The methods I briefly describe here can be applied in an organized professional environment in order to estimate and expose the parametric configuration of any slot game whose PAR sheet is missing.

In the next sections, I use the same denotations as used in the section The Parametric Configuration of Slot Machines as a Base for the Probabilistic Models for Slot Games.

#### **The Raw Approximation**

This method is based on the well-known result from probability theory called Bernoulli's theorem, which states that in a sequence of independent experiments performed under identical conditions, the sequence of the relative frequencies of the occurrence of an event is convergent toward the probability of that event.

Applied to slots, that principle says that if  $N$  is the number of spins of a reel with  $t$  stops, where we observe as an outcome a specific symbol  $S$  that is placed on  $c$  stops, and  $n(N)$  is the number of occurrences of  $S$  after the  $N$  spins, then the sequence  $(n/N)_N$  is convergent toward the probability of occurrence of  $S$ , namely  $P(S) = c/t$ .

The ratio  $n/N$  is the relative frequency of occurrence of  $S$ . It follows that for large values of  $N$ , the relative frequency of occurrence of  $S$  approximates the probability of  $S$  occurring. The higher the  $N$ , the more accurate this approximation. Obviously, the number of spins  $N$  must be large enough to obtain good approximations of the ratios  $c_i/t$ , and this is the main issue of this method. As theory does not provide us with tools for choosing  $N$  for a given error range, all we have is the principle "the larger the  $N$ , the better".

As one can notice, this method of approximation, which is based on statistical observation, is subject to errors coming from idealizations and various assumptions,

and the error ranges are not even quantifiable. Given these issues, the best way to use this method is not for individual records, but for cumulative records coming from several sources and for refining the estimations in correlation with the increase in the total number of spins  $N$ . This principle is also common for the odds calculators based on partial simulations, which are used for various games.

Note that the described method provides us with approximations of the ratios  $c_i/t$  (the basic probabilities) for each reel and not the parameters of the configuration individually ( $c_i$  and  $t$ ). However, knowing the basic probabilities is enough for any probability computation for a slot game, as seen in the section The Parametric Configuration of Slot Machines as a Base for the Probabilistic Models for Slot Games.

A more accurate approximation of the ratios  $c_i/t$  and even of  $c_i$  and  $t$  individually is still possible through statistical observation, using a method that can refine the raw estimations obtained through the previously described method. Such a method is briefly described in the next section.

### Denominator-Match Method

Denote by  $n_1(N), n_2(N), \dots, n_p(N)$  the number of occurrences of symbols  $S_1$  to  $S_p$ , respectively, after ( $N$ ) spins of a reel. There is a slight correlation between the recorded values  $n_1, n_2, \dots, n_p$  for various large numbers of spins ( $N$ ). From this correlation, we can refine the estimation of the ratios  $c_i/t$  obtained through the previous method and also find estimations for  $c_1, c_2, \dots, c_p$  and ( $t$ ), by recognizing a numerical pattern across some sequences of fractions representing the ratios between possible values for  $c_i$  and ( $t$ ).

The denominator-match method is based on the numerical analysis of the fractions  $n_i/N$  and on a five-step algorithm briefly explained below.

We write each fraction  $n_i/N$  as a chain of equal fractions, having numerators from 1 upward and denominators that are not necessarily integers, for every  $i$  from 1 to  $p$ . Across the  $p$  chains of equal fractions obtained, we choose that of minimal length (let  $m$  be the minimal length). Then, across the  $p$  chains of equal fractions, we extract  $m$  sequences of fractions (one fraction from each equality chain) having the denominators nearest to the denominators from the minimal equality chain. From the  $m$  sequences of fractions obtained, we choose one sequence of  $p$  fractions by applying progressively the following filtering criteria: the denominators are as close to each other as possible, there is the highest number of instances of the same denominator, and the repeating denominator with the largest share is an integer. As a final step, we adjust the numerators of the final sequence of fractions as follows: If the sum of the numerators lies between the minimum and maximum of the denominators, then we take the numerators as the symbol distribution on the reel



$(c_i)$  and their sum as the number of stops of the reel ( $t$ ); if their sum does not lie in that interval, then through addition or subtraction, we distribute, proportionally with their values, the difference between their sum and the integer nearest to the mean of the minimal and maximal denominator, rounding the added/subtracted quantities to integers. For our resulting estimation, we take the adjusted numerators as the symbol distribution on the reel ( $c_i$ ) and the integer nearest to the mean of the minimal and maximal denominators as the number of stops of the reel ( $t$ ).

This method provides us with the most probable number of stops  $t$  and associated symbol distribution  $c_1, c_2, \dots, c_p$  of a reel in a certain probability field; the error range of this approximation is quantifiable in terms of probability (Bărboianu, 2013b).

Regarding the practical application of the methods through statistical observation, it is obviously an arduous task, since we have to watch and record spins in the numbers of thousands. For online games, software can be developed to help in such an endeavor. For physical machines, it is far more difficult to watch and note thousands of outcomes just for one reel of a machine, not to mention that the slots operator might not allow this action. Of course, technology based on video capturing might help with such a task, but that is not the concern of the current study.

### Physical Measurements

Any information acquired on ( $t$ ) besides the presented statistical methods of estimation is useful with respect to the accuracy of the approximations because it can provide a clue as to how high we should choose ( $N$ ) in order to avoid irrelevant results (for example, if ( $t$ ) = 100, we intuit that choosing ( $N$ ) = 1,000 or lower would not be high enough for relevant results).

Besides the methods based on statistical observation, there exists a method of estimating  $t$  through physical measurements, applicable to some particular types of slot machines. This method exploits the information given by the appearance of the reel on the display. As we know, only a small part of the reel (either physical or virtual) is visible on the display and this part can be seen as one or several adjacent symbols (usually three, but up to five). Thus, we can view from one to five consecutive symbols of the reel. If the appearance of this part of the reel is three-dimensional (which is possible for both physical and virtual reels), by measuring some parameters of this image, we can deduce an estimation for the number of stops of that reel ( $t$ ). Basically, the apparent lengths of the visible symbols give full information on the curvature of the reel, which then leads to an estimation of the entire number of stops, since the number of visible symbols per the circular length of the visible reel is proportional to the total number of stops per the circular length of the entire reel (Bărboianu, 2013c). This method can be applied only to reels showing at least two consecutive symbols on the display in three-dimensional view. The method cannot be applied to virtual reels showing

several consecutive symbols in a flat image. As in the case of the previous method based on statistical observation, there are issues with the practical application of the method based on physical measurements. There might be technical issues regarding acquiring the proper position for measurement or placing the measurement tool on the surface of the machine. In addition, for this method, an alternative would be for the observer to take photos and make measurements on the photos. Of course, the slot machine operator might not allow direct measurement and/or taking of photos.

With this incursion into the mathematical methods of approximating the parameters of the configuration of a slot machine through statistical observation, I conclude the analysis of the possible reasons for slot producers keeping their PAR sheets secret. Summarizing below the arguments against the justifications for these possible reasons, I draw the conclusion that the secrecy of slot producers on the parametric configuration of their slot machines is not rationally justified:

- Protection against competition fails against the generality of the math formulas and equations and the open possibility for all slot producers to configure any parametric design for their slot machines and in so doing, manipulating the game parameters and the payout schedule in unlimited ways, so as to obtain the desired statistical indicators for the house.
- The fear of losing players who face the real odds of winning attached to their games, thereby affecting the popularity of slot games, fails against the a priori expectation of the players for low and very low odds of winning induced by the experienced secrecy of PAR sheets and against the lottery example, in which lottery players continue to play against the lowest odds of winning because of other addictive elements, which slots also hold.
- The secrecy itself fails against the statistical methods that mathematics provides us with for retrieving the missing data through statistical observation, even as approximated results.

### **The Exposure of the Parametric Configuration and the Mathematical Facts of a Game as an Ethical Obligation**

The requirement for the exposure of the parametric configuration applies only to slots, since it is the only existent game of chance for which such data are hidden. Indeed, while a slot machine displays only a part of each reel in the stop position, for the other games, all of the configuration from which the outcome is produced is visible for the players—the roulette numbers are shown on the wheel and table, the deck composition is known for every card game, dice faces and the number of dice are visible for every dice game, lottery numbers are known for each lottery design, and so forth.

The information to be exposed would be in the form of a technical/mathematical sheet specific to each slot game, either consisting only of the parametric

configuration, or of the parametric configuration plus basic mathematical results such as the probabilities of the winning combinations shown on the payout schedule, probability of any win, and expected value. For the former variant, which is merely informative and provided either by the slot producer or retrieved through the methods I described in the previous section, it would remain for the player to inquire further for the mathematical results as an optional action. For the latter variant, the probability/statistics part that comes along with the parametric configuration would be completed by an assigned mathematical authority.

The goal of exposing parametric configurations for slot games is not so much to place slots in the same status as other games of chance, as it is a matter of ethics.

The exposure of the parametric configuration of a game to the player prior to playing is an ethical obligation in two ways—one commercial and the other humanitarian.

The commercial way treats the game as any commercial service, for which full technical specifications are required from the producer to the customer. Just as a public coffee machine must show the coffee brand name and the coffee volume returned for the unit price, so the slot machine must show the reels' numbers of stops and the symbol weighting; a bet is still a purchased service once the player inserts a non-returnable coin in the machine. A relevant example of unethical procedures would be having identical-looking machines yielding different payback percentages as a result of different (missing) parametric configurations or even the same machine changing its payback percentage with the replacement of a single chip.

The humanitarian way is related first, to the free will of thought and second, to the limitation of addiction risk. Being informed on all parameters of a game one plays is a condition for unconstrained (constrained through omission) personal thinking leading to personal actions. It is as though someone asks you to bet that you can jump from a high place and land on your feet; if you know in advance the height from which you will jump, or if you measure it before you bet, you might decline the bet or propose another one for a certain measurement, and this implies a free decision. Such a comparison can also be an argument for the skeptical slot player who could ask: "Given that slots is not a strategic game played against opponents (as with poker, for instance, where the odds are essential in evaluating the advantage in a given situation), why do we need 'all the math stuff' associated with slots?" The answer is simple: information and strategy. The argument for information is expressed through the preceding comparison. Regarding strategy, in slots there is a trivial strategy, namely, the strategy of choosing: choosing one game or another, choosing how many paylines to enable, choosing the parameters of time and money management, and of course, choosing to quit one game for another or choosing not to play at all. The only objective criteria for such a strategy are probabilities and expected value. Similar comparisons that also hold in some ethical aspects are the

illness/danger warnings on cigarette packs and the “possible adverse effects” statistics in drug leaflets.

Regarding the limitation of addiction risk, past and ongoing studies debating the issue of whether mathematical knowledge (provided either as pre-calculated results such as winning odds and other statistical indicators, or in the form of learning theoretical and applied probability theory basics) causes a decrease in gambling behavior have not yet reached a clear conclusion. Several empirical studies found no significant changes in college students’ gambling activity after they received a didactic intervention on gambling mathematics (Hertwig, Barron, Weber, & Erev, 2004; Steenbergh, Whelan, Meyers, May, & Floyd, 2004; Williams & Connolly, 2006). On the other hand, more theoretical studies proved that postsecondary statistics education developed critical thinking, which also applies to gambling, and the gamblers who receive such education tend to have significantly lower rates of problem gambling (Abbott & Volberg, 2000; Gerstein et al., 1999; Gray & Mill, 1991). I am personally inclined to think that such a decrease follows optimal mathematical learning, which can be devised and developed according to its scope, and this will be the focus of forthcoming research.

Given the ethical obligation to expose the parametric configuration of the slot games, the question arises as to how this information can be technically exposed. With its relation to risk factors, exposure on a website would not be enough, because in a physical casino or slot room, there are specific physical addictive elements that might distract a player’s mind from the mathematical facts seen earlier on the Internet, not to mention that the player might encounter a slot machine for which he or she had not studied its technical/mathematical sheet beforehand. It follows that the technical/mathematical sheet must accompany each slot machine in printed form or at least be available upon request from the slot operator. Online sheets are applicable to the online slot games. Since slot operators, like slot producers, might consider that it is not to their advantage to provide the technical/mathematical sheets to their customers, such an action is impossible only by law, which can also certify an official authority to provide the mathematical facts of the games.

The debate remains open as to whether the technical/mathematical sheets must contain only the parametric configuration (as sufficient information for someone to optionally compute further mathematical results), or, in addition, basic mathematical results concerning probabilities and other statistical indicators with respect to the ethical requirement. The former alternative raises the question of the usefulness of mathematical didactical intervention for gamblers, and both alternatives raise the question of understanding and interpretation of the exposed or learned mathematical concepts and facts related to games of chance, since the simple acquisition of numerical probabilities and statistical indicators as mere quantities might not be enough for the decisions made based upon them. These issues will be treated in a

forthcoming article, as conditions for an optimal mathematical didactical intervention in gambling.

### Acknowledgements

I want to thank to the reviewer who suggested relevant examples for the hypothetical situations presented in the paper: the start-up company example within the competitive-prejudice counter-argument and the example of two identical-looking slot machines having different parametric configurations within the ethical argument.

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Manuscript history: submitted March 13, 2013; accepted April 29, 2014. This article was peer-reviewed. All URLs were available at the time of submission.

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Competing interests: None declared (all authors).

Ethical approval: Not required.

Funding: None.

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